LXXIII. *Of the beſt Form of Geographical Maps. By the Rev.* Patrick Murdoch, *M. A. F. R. S.*

Read Feb. 9, 1758.

I. WHEN any portion of the earth's ſurface is projected on a plane, or transferred to it by whatever method of deſcription, the real dimenſions, and very often the figure and poſition of countries, are much altered and miſrepreſented. In the common projection of the two hemiſpheres, the meridians and parallels of latitude do indeed interſect at right angles, as on the globe; but the linear diſtances are every-where diminiſhed, excepting only at the extremity of the projection: at the center they are but half their juſt quantity, and thence the ſuperficial dimenſions but one-fourth part: and in leſs general maps this inconvenience will always, in ſome degree, attend the *ſtereographic* projection.

The *orthographic*, by parallel lines, would be ſtill leſs exact, thoſe lines falling altogether oblique on the extreme parts of the hemiſphere. It is uſeful, however, in deſcribing the circum-polar regions: and the rules of both projections, for their elegance, as well as for their uſes in aſtronomy, ought to be retained, and carefully ſtudied. As to Wright's, or Mercator's, nautical chart, it does not here fall under our conſideration: it is perfect in its kind; and will always be reckoned among the chief inventions of the laſt age. If it has been miſunderſtood, or miſapplied, by geographers, they only are to blame.

II. The

II. The particular methods of deſcription propoſed or uſed by geographers are ſo various, that we might, on that very account, ſuſpect them to be faulty; but in moſt of their works we actually find theſe two blemiſhes, *the linear diſtances viſibly falſe*, and *the interſections of the circles oblique*: ſo that a quadrilateral rectangular ſpace ſhall often be repreſented by an oblique-angled rhomboid figure, whoſe diagonals are very far from equal; and yet, by a ſtrange contradiction, you ſhall ſee a fixed ſcale of diſtances inſerted in ſuch a map.

III. The only maps I remember to have ſeen, in which the laſt of theſe blemiſhes is removed, and the other leſſened, are ſome of P. Schenk's of Amſterdam, a map of the Ruſſian empire, the Germania Critica of the famous Profeſſor Meyer, and a few more ‡. In theſe the meridians are ſtraight lines converging to a point; from which, as a center, the parallels of latitude are deſcribed: and a rule has been publiſhed for the drawing of ſuch maps *. But as that rule appears to be only an eaſy and convenient approximation, it remains ſtill to be inquired, *What is the conſtruction of a particular map, that ſhall exhibit the ſuperficial and linear meaſures in their trueſt proportions?* In order to which,

IV. Let E*l*LP, in this figure (*See* TAB. XXI.) be the quadrant of a meridian of a given ſphere, whoſe center is C, and its pole P; EL, E*l*, the latitudes of two places in that meridian, EM their

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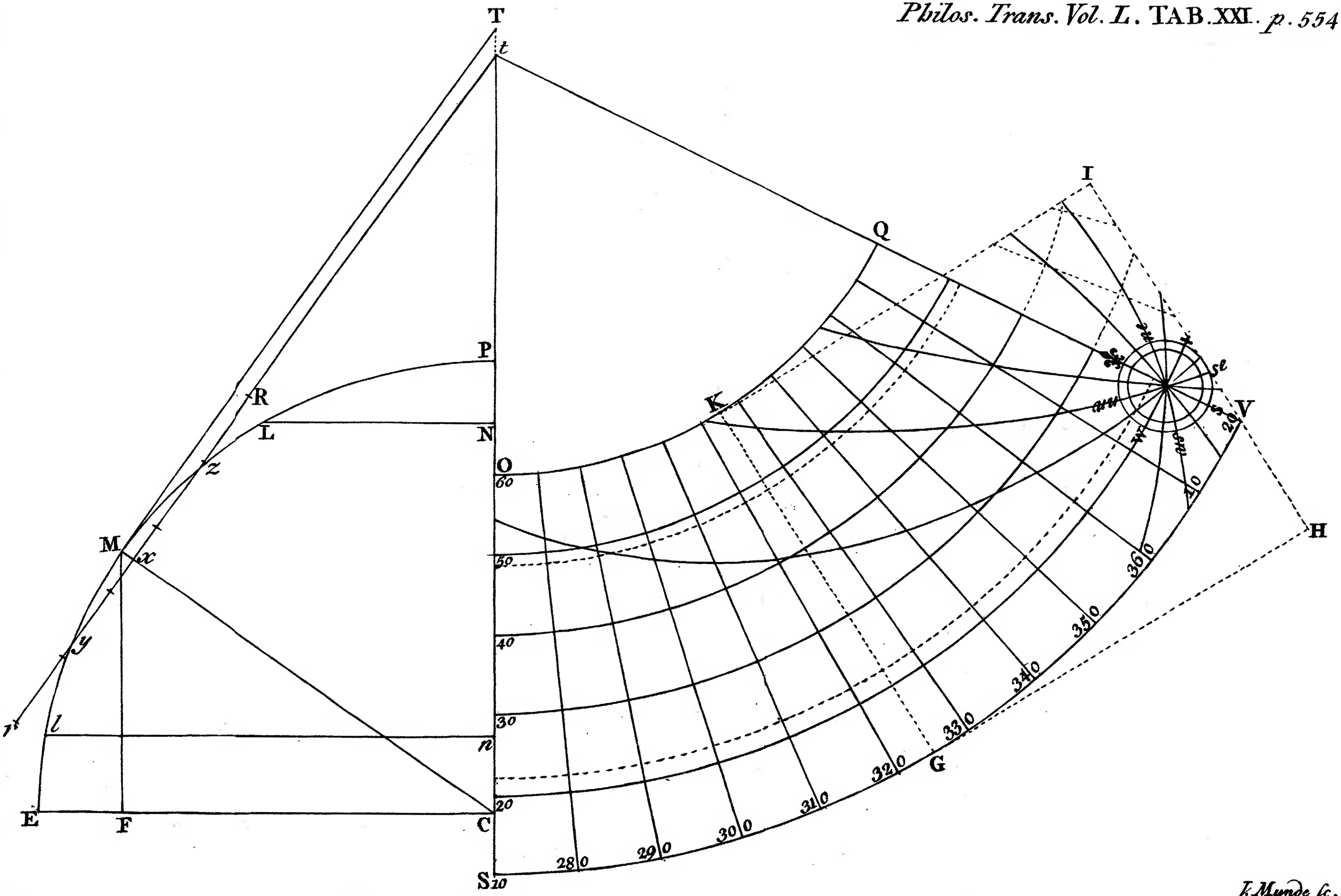
‡ *Senex* drew ſeveral of that form.

* See the Preface to the ſmall Berlin Atlas.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middle latitude. Draw LN, ln, coſines of the latitudes, the ſine of the middle latitude MF, and its cotangent MT. Then writing unity for the radius, if in CM we take $Cx = \frac{Nn}{Ll \times MF \times MT}$, and thro' x we draw xR, xr, equal each to half the arc Ll, and perpendicular to CM; the conical ſurface generated by the line Rr, while the figure revolves on the axis of the ſphere, will be equal to the ſurface of the zone that is to be deſcribed in the ſame time by the arc Ll; as will eaſily appear by comparing that conical ſurface with the zone, as meaſured by *Archimedes*.

And, laſtly, If from the point t, in which rR produced meets the axis, we take the angle CtV in proportion to the longitude of the propoſed map, as MF the ſine of the middle latitude is to radius, and draw the parallels and meridians as in the figure, the whole ſpace SOQV will be the propoſed part of the conical ſurface expanded into a plane; in which the places may now be inſerted according to their known longitudes and latitudes.

EXAMPLE.

V. Let Ll, the breadth of the zone, be 50°, lying between 10° and 60° north latitude; its longitude 110°, from 20° eaſt of the Canaries to the center of the weſtern hemiſphere; comprehending the weſtern parts of Europe and Africa, the more known parts of North America, and the ocean that ſeparates it from the old continent.

And becauſe $Cx = \frac{Nn}{Ll \times MF \times MT}$, add theſe three logarithms.

Log.

Log. 0.8726650 (= 50° to radius 1) — 1.9408476
Log. MF (ſin. 35°) . . . . — 1.7585913
Log. MT (tang. 55°) 0.1547732

Take the ſum . . . . — 1.8542121
from log. Nn (= .6923772) . . . . — 1.8403427

the remainder . . . . — 1.9861306
is the logarithm of Cx. And becauſe 1 :
Cx : : MT : xt, to this adding the log. MT 0.1547732

The ſum 0.1409038
is the log. of xt = 1.383260; and xR (= xr = ½ Ll) being .4363325, Rt will be 0.9469275, rt = 1.8195925. Whence having fixed upon any convenient ſize for our map, the center t is eaſily found. As, allowing an inch to a degree of a great circle, or 50 inches to the line Rr, Rt the ſemidiameter of the leaſt parallel will be 54.255 inches, and that of the greateſt parallel 104.255 inches.

Again, making as radius to MF ſo the longitude 110° to the angle StV, that angle will be 63° 5′ $\frac{3}{5}$. Divide the meridians and parallels, and finiſh the map as uſual.

> *Note*, The log. MT being repeated in this computation with a contrary ſign, we may find xt immediately by ſubtracting the ſum of the logarithms of Ll and MF from the log. of Nn.

VI. A map drawn by this rule will have the following properties:

1. The interſections of the meridians and parallels will be rectangular.

2. The

2. The diſtances north and ſouth will be exact; and any meridian will ſerve as a ſcale.

3. The parallels thro' z and y, where the line Rr cuts the arc Ll, or any ſmall diſtances of places that lie in thoſe parallels, will be of their juſt quantity. At the extreme latitudes they will exceed, and in mean latitudes, from x towards z or y, they will fall ſhort of it. But unleſs the zone is very broad, neither the exceſs nor the defect will be any-where conſiderable.

4. The latitudes and the ſuperficies of the map being exact, by the conſtruction, it follows, that the exceſſes and defects of diſtance, now mentioned, compenſate each other; and are, in general, of the leaſt quantity they can have in the map deſigned.

5. If a thread is extended on a plane, and fixed to it at its two extremities, and afterwards the plane is formed into a pyramidal or conical ſurface, it may be eaſily ſhewn, that the thread will paſs thro' the ſame points of the ſurface as before; and that, *converſely*, the ſhorteſt diſtance between two points in a conical ſurface is the right line which joins them, when that ſurface is expanded into a plane. Now, in the preſent caſe, the ſhorteſt diſtances on the conical ſurface will be, if not equal, always nearly equal, to the correſpondent diſtances on the ſphere: and therefore, all rectilinear diſtances on the map, applied to the meridian as a ſcale, will, nearly at leaſt, ſhew the true diſtances of the places repreſented.

6. In maps, whoſe breadth exceeds not 10° or 15°, the rectilinear diſtances may be taken for ſufficiently exact. But we have choſen our example of a greater breadth than can often be required, on purpoſe

poſe to ſhew how high the errors can ever ariſe; and how they may, if it is thought needful, be nearly eſtimated and corrected.

Write down, in a vacant ſpace at the bottom of the map, a table of the errors of equidiſtant parallels, as from five degrees to five degrees of the whole latitude; and having taken the mean errors, and diminiſhed them in the ratio of radius to the ſine of the mean inclination of the line of diſtance to the meridian, you ſhall find the correction required; remembering only to diſtinguiſh the diſtance into its parts that lie *within* and *without* the ſphere, and taking the difference of the correſpondent errors, in *defect* and in *exceſs*.

But it was thought needleſs to add any examples; as, from what has been ſaid, the intelligent reader will readily ſee the uſe of ſuch a table; and chiefly as, whenever exactneſs is required, it will be more proper, and indeed more expeditious, to compute the diſtances of places by the following canon.

Multiply the product of the coſines of the two given latitudes by the ſquare of the ſine of half the difference of longitude; and to this product add the ſquare of the ſine of half the difference of the latitudes; the ſquare root of the ſum ſhall be the ſine of half the arc of a great circle between the two places given.

Thus, if we are to find the true diſtance from one angle of our map to the oppoſite, that is, from S to Q, the operation will be as follows:

L. ſin. 30° = — 1.6989700
L. ſin. 80° = — 1.9933515
2 L. ſin. 55° = — 1.8267290
— 1.5190505 = log. of 0.330408
and 2 L. ſin. 25° = — 1.2518966 = log. of 0.178606
Log. of the ſum . . . 0.509014 is — 1.7067297
Whole half is — 1.8533648
the L. ſin. of 45° 31′, the double of which is 91° 2′, or 5462 geographical miles.

And ſeeing the lines TS, TQ, reduced to minutes of a degree, are 6255.189 and 3255.189 reſpectively. and the angle STV is $63^\circ\ 5'\frac{3}{7}$, the right line SQ on the map will be 5594′, exceeding its juſt value by 132′ or $\frac{1}{42}$ of the whole.

7. The errors on the parallels increaſing faſt towards the north, and the line SQ having, at laſt, nearly the ſame direction, it is not to be wondered that the errors in our example ſhould amount to $\frac{1}{42}$. Greater ſtill would happen, if we meaſured the diſtance from O to Q by a ſtraight line joining thoſe points: for that line, on the conic ſurface, lying every-where at a greater diſtance from the ſphere than the points O and Q, muſt plainly be a very improper meaſure of the diſtance of their correſpondent points on the ſphere. And therefore, to prevent all errors of that kind, and confine the other errors in this part of our map to narrower bounds, it will be beſt to terminate it towards the pole by a ſtraight line KI touching the parallel OQ in the middle point K, and on the eaſt and weſt by lines, as HI, parallel to the meridian thro' K, and meeting the tangent at the middle point of the parallel SV in H. By this means too we ſhall gain more ſpace than we loſe, while the map takes the uſual rectangular

rectangular form, and the ſpaces GHV remain for the *title*, and other inſcriptions.

VII. Another, and not the leaſt conſiderable, property of our map is, that it may, without ſenſible error, be uſed as a ſea-chart; the rumb-lines on it being logarithmic ſpirals to their common pole t, as is partly repreſented in the figure: and the arithmetical ſolutions thence derived will be found as accurate as is neceſſary in the art of ſailing.

Thus if it were required to find the courſe a ſhip is to ſteer between two ports, whoſe longitudes and latitudes are known, we may uſe the following

RULE.

To the logarithm of the number of minutes in the difference of longitude add the conſtant logarithm * — 4.1015105, *and to their ſum the logarithm ſine of the mean latitude, and let this laſt ſum be* S.

The cotangent of the mean latitude being T, *and an arithmetical mean between half the difference of latitude and its tangent being called* m, *from the logarithm of* T + m *take the logarithm of* T — m, *and let the logarithm of their difference be* D; *then ſhall* S — D *be nearly the logarithm tangent of the angle, in which the ſhip's courſe cuts the meridians.*

Note, We ought, in ſtrictneſs, to uſe the ratio of $tx + x\mathrm{R}$ to $tx - x\mathrm{R}$ inſtead of $\mathrm{T} + m$ to $\mathrm{T} - m$; but we ſubſtitute this laſt as more eaſily computed, and very little different.

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* This conſtant logarithm contains the reduction of the diff. of longitude to parts of radius unity, and to *Briggs*'s Modulus.

EXAMPLE

EXAMPLE 1.

Let the latitudes, on the ſame ſide of the equator, be 10° and 60°; then the middle latitude and its complement are 35° and 55°, and half the difference of the latitudes is 25°: and the difference of longitude being 110°, the operation will ſtand as below.

Log. 6600′ (in 110°) . . 3.8195439
Conſtant log. . . . — 4.1015105

— 1.9210544
Log. ſin. 35° . . . — 1.7585913

S = ... — 1.6796457

Again T = 1.4281480
m = .4513202

Log. $\overline{T+m}$ (= 1.8794682) 0.2740350
Log. $\overline{T-m}$ (= 0.9768278) — 1.9898180

Log. 0.2842170 = D = — 1.4536500

S — D (= log. tangent 59° 16′) . . . . . = 0.2259957
agreeing to a minute with the ſolution by a table of meridional parts.

EXAMPLE 2.

The reſt remaining, let the difference of longitude be only 40°; then

Log. 2400′ (in 40°) . . 3.3802112
Conſtant log. . . . — 4.1015105

— 1.4817217
Log. ſin. 35° . . . — 1.7585913

S = ... — 1.2403130
D (as before) = — 1.4536500

S — D (= log. tang. 31° 27′½) . . . — 1.7866630

EXAMPLE 3.

Let the difference of longitude be 40°; but the latitudes 56° and 80°;

$$\left.\begin{array}{l}\text{And log. } 2400' \\ +\ \text{log. conſtant}\end{array}\right\} = -\ 1.4817217$$

$$\text{Log. ſin. } 68^\circ\ .\ .\ . = -\ \underline{1.9671659}$$

$$S = .\ .\ .\ .\ -\ 1.4488876$$

$$T\ (\text{tang. } 22^\circ) = .4040262$$

$$m = .\ .\ .\ .\ .\ .\ \underline{.2109980}$$

$$\text{Log. } \overline{T+m}\ (= .6150242) -\ 1.7888921$$

$$\text{Log. } \overline{T-m}\ (= .1830282) -\ \underline{1.2625181}$$

$$\text{Log. . . } \overline{0.5263740} = D = -\ \underline{1.7212944}$$

$$S - D\ (= \text{log. tangent } 28^\circ\ 6')\ .\ .\ .\ .\ .\ .\ .\ . = -\ 1.7275932$$

wanting of the true anſwer no more than 1° 4′.

And in all caſes that can occur, the error of this rule will be inconſiderable.

It is not meant, however, that it ought to take place of the eaſier and better computation by a table of meridional parts: but it was thought proper to ſhew, by ſome examples, how ſafely the map itſelf may be depended on in the longeſt voyages; provided it is ſufficiently large, and the neceſſary rumb-lines are exactly drawn *.

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* See *Coteſii* Logometr. prop. 6.

LXXIV.

“ laſt age. If it has been miſunderſtood or miſapplied by geographers, they only are to blame.”— And again, at the end of his nautical examples, he concludes thus, *viz.* “ It is not meant, however, “ that it ought to take place of the eaſier and “ better computation by a table of meridional “ parts.”

I have the honour to be, with the greateſt reſpect,

SIR,

The ROYAL SOCIETY’s, and

Your moſt obedient Servant,

William Mountaine.

ADDENDA *to Mr.* Murdoch*'s Paper*, N°. LXXIII.

IF it is required “ to draw a map, in which the ſuperficies of a “ given zone ſhall be equal to the zone on the ſphere, while “ at the ſame time the projection from the center is ſtrictly geometrical ;” *Take* Cx *to* CM *as a geometrical mean between* CM *and* N n, *is to the like mean between the coſine of the middle latitude, and twice the tangent of the ſemidifference of latitudes*; and project on the conic ſurface generated by *xt*. But here the degrees of latitude towards the middle will fall ſhort of their juſt quantity, and at the extremities exceed it: which hurts the eye. Artiſts may uſe either rule: or, in moſt caſes, they need only make C*x* to CM as the arc ML is to its tangent, and finiſh the map; either by a projection, or, as in the firſt method, by dividing that part of *xt* which is intercepted by the ſecants thro’ L and *l*, into equal degrees of latitude.

Mr. Mountaine juſtly obſerves, “ that my rule does not admit of “ a zone containing N. and S. latitudes.” But the remedy is, *to extend the leſſer latitudes to an equality with the greater; that the cone may be changed into a cylinder, and the rumbs into ſtraight lines.*